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Computer Simulation of Selective Excitation in n.m.r. Imaging

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Computer simulation of selective excitation in n.m.r. imaging

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It is attractive to apply selective excitation as part of a method for proton spin imaging of large objects. In the past, there has been some discussion on how selective excitation should be performed. The spin magnetization reacts in a nonlinear way to an r.f. (radio frequency) excitation pulse. This makes it difficult to give algebraic expressions for the excited transverse magnetization if one does not apply just a simple rectangular r.f. pulse but, instead, an arbitrarily tailored pulse. We have, therefore, studied numerical solutions of the equations of motion (Bloch equations) for various forms of tailored pulses. Typical computation results will be shown and a brief discussion on practical aspects will be given.

1. Introduction

Selective excitation is an essential part of several spin imaging methods. When the task is to make a good n.m.r. picture of just one slice, it seems to us advantageous to excite selectively the nuclei of that slice only. Excitation of the other nuclei and subsequent efforts to get rid of their signals may give rise to extra noise and will, at least, make greater demands on the receiving system with respect to the dynamic range. This is especially so if one has to deal with a large object, such as the human body.

Since 1973 (Tomlinson & Hill 1973) the subject of selective exictation has been widely discussed in the literature on high resolution n.m.r. An extensive treatment was recently given by Morris & Freeman (1978). In spin imaging, however, the experimental conditions are rather different, since one has to deal with much shorter times and much larger samples. As a result, one usually cannot start the detection until the excitation is completed. The discussion in this paper deals with this situation.

Ideas on applying selective excitation in spin imaging methods have been appearing in literature since 1974 (Garroway *et al.* 1974; Lauterbur *et al.* 1974), but important practical details have not been clear. Early in 1978 two papers (Sutherland & Hutchison 1978; Hutchison *et al.* 1978) appeared that gave good details of a proposed experimental procedure. Some results of a computer simulation were given as well, namely those for a Gaussian-shaped amplitude modulation of the r.f. excitation pulse and those for a modulation function that is the product of a Gauss function and a (sin t)/t function. It is essential to apply a means of time reversal, for instance a gradient reversal, to bring the excited spins back into phase (echo formation). This was also proposed by Hoult (1977), who has criticized the earlier descriptions of selective excitation, in which echo formation is not part of the procedure.

This controversial matter is further discussed in two papers that are about to be published, namely those by Mansfield *et al.* (1979) and by Hoult (1979). Because of the unclear situation, we have written computer programs for a simulation study.

Up to now we have studied only amplitude modulation of the r.f. excitation, but a study of phase modulation would also be an interesting task. In view of the limited length of the present paper, we shall consider only two modulation functions, namely, in § 2, a rectangular time function and, in § 3, the product of a $(\sin t)/t$ function and a relatively broad Gauss function, the Fourier transform of which is a rounded rectangle in frequency space. The results can be considered as an extension of the work of Hutchison *et al.* (1978). We shall further compare these results with Hoult's (1979) perturbation theory. Finally, some consequences for a practical realization will be considered in § 4.

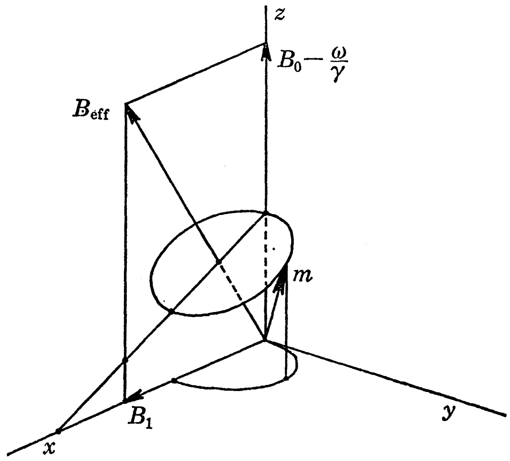

FIGURE 1. Rotating frame used to describe the motion of the magnetization, $\boldsymbol{m}$.

2. Rectangular time function

For a certain time, typically of the order of 100 μs, a circularly polarized r.f. field, $\boldsymbol{B}_1$, of constant angular frequency, ω, and of constant amplitude, B_1, is applied to the sample. In addition, a strong static magnetic field, $\boldsymbol{B}_0$, is present; this field is constant during the r.f. pulse and points in the z direction, perpendicularly to the plane of $\boldsymbol{B}_1$. Its magnitude, B_0, is at least 10^{-1} T and has a small spatial variation (gradient) of the order of 10^{-3} T over the entire sample, which gives various parts different resonance frequencies in the immediate vicinity of $\omega/2\pi$. B_1 is, typically, of the order of 10^{-4} T, or somewhat lower.

Figure 1 shows the familiar rotating frame, rotating about z at angular frequency ω. It is used to describe the motion of the magnetization vector, $\boldsymbol{m}$, of a group of proton spins. The r.f. field, $\boldsymbol{B}_1$, points constantly along the x coordinate of the rotating frame.

In figure 1 we consider the magnetization, $\boldsymbol{m}$, of those spins that experience the indicated value of $B_0 - \omega/\gamma$ and thus a total field $\boldsymbol{B}_{eff}$. At the start of the r.f. pulse the end point of $\boldsymbol{m}$ lies on the z axis. During the pulse, it moves along the circle indicated, of which the projection onto the xy plane is partially drawn.

Relaxation times are assumed to be infinitely long, so that the length, m, of the vector $\boldsymbol{m}$ is constant during the motion.

For spins resonating at precisely ω $(B_0 = \omega/\gamma)$ the field $\boldsymbol{B}_{eff}$ coincides with $\boldsymbol{B}_1$ and the circle lies in the yz plane. For this instance the angle through which $\boldsymbol{m}$ is rotated during the rectangular r.f. pulse is called θ. For the familiar instance of $\theta = 90°$ it should be noted that the end point of $\boldsymbol{m}$ makes several revolutions along the circle, if $|B_0 - \omega/\gamma| \gg B_1$, while m_y and m_x are not yet negligible.

With use of figure 1 it is easy to derive the following exact expressions for the magnetization components at the point in time at which the rectangular r.f. pulse ends:

$$m_y/m = (\sin \theta g)/g, \tag{1a}$$

$$m_x/m = f(1-\cos\theta g)/g^2, \quad (1b)$$

and

$$m_z/m = (f^2+\cos\theta g)/g^2, \quad (1c)$$

where

$$f = (B_0 - \omega/\gamma)/B_1,$$
$$g = (1+f^2)^{\frac{1}{2}},$$

and θ = flip angle at $f = 0$.

The three equations (1) are not independent since

$$m_x^2 + m_y^2 + m_z^2 = m^2.$$

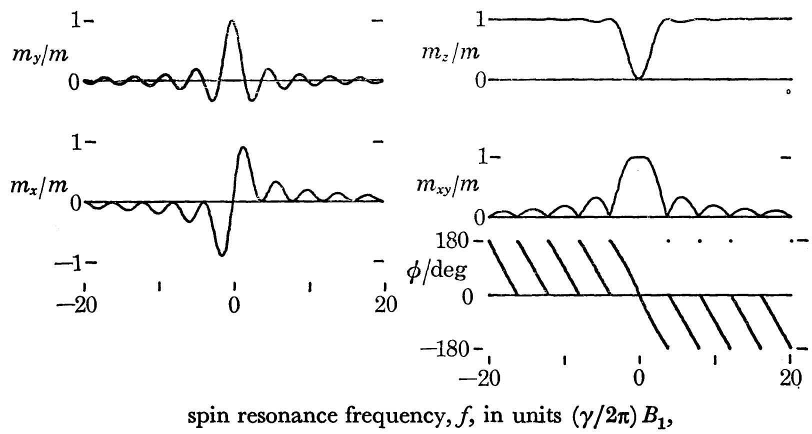

FIGURE 2. Magnetization components at the point in time at which a rectangular excitation ends. The curves are drawn for $\theta(f=0) = 90°$ and show the dependence on the spin resonance frequency, f, after quadrature phase-sensitive detection.

For $\theta = 90°$ the components m_y, m_x, and m_z are plotted in figure 2 as a function of f, which is the spin resonance frequency in units of $(\gamma/2\pi)B_1$, after quadrature phase-sensitive detection with reference $\omega/2\pi$. The magnetization component in the xy plane is also given, by means of the polar coordinates m_{xy} and ϕ, where $m_y = m_{xy}\cos\phi$ and $m_x = -m_{xy}\sin\phi$.

In a normal experiment one detects the components m_y and m_x. The expressions for these, equations (1a) and (1b), have already been given by Mansfield *et al.* (1979). These authors have discussed at same length this instance of a rectangular r.f. pulse and have illustrated it by experiments. They conclude that a selective excitation is obtained, but we would like to stress the fact that spins relatively far outside the central frequency region are also excited in a way that cannot be disregarded, as one can see from figure 2. There is no way to avoid this. The reason is in the steep rise and fall of the rectangular excitation.

A better selection will be obtained by means of the excitation discussed in § 3, where we describe a computer calculation to find the result of the excitation. The instance of a rectangular excitation was treated because of its mathematical simplicity, and it already reveals some essential features, such as phase deviations of several times 360° at the end of the pulse.

3. Tailored time function

The r.f. amplitude, B_1, is no longer taken to be constant during the r.f. pulse. For presentation in this paper we have chosen a typical example, namely the following function of the time t:

$$\left.\begin{aligned} B_1(t) &= B_1(0)\,\mathrm{sinc}\,(2t/p)\exp(-t^2/2d^2), \quad \text{for} \quad -4p < t < 4p,\\ B_1(t) &= 0, \quad \text{for} \quad |t| \geqslant 4p, \end{aligned}\right\} \quad (2)$$

where
$$d/p = 95/64,$$
and
$$\mathrm{sinc}\,(x) \equiv (\sin \pi x)/\pi x \quad \text{(Bracewell 1965)}.$$

A sketch of this function is given in figure 3. Its Fourier transform is given for later comparison with the excited magnetization as a function of the spin resonance frequency. This Fourier transform is known to be a convolution of a rectangle of half width $1/p$ and a Gauss function with bending-point half width $32/95\pi p = 0.10722 p^{-1}$.

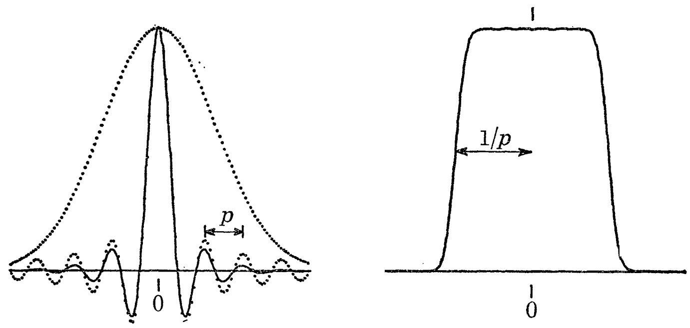

FIGURE 3. Tailored excitation function (left) and its Fourier transform (right). The dotted lines give the sinc and the Gauss function, of which the excitation function is the product.

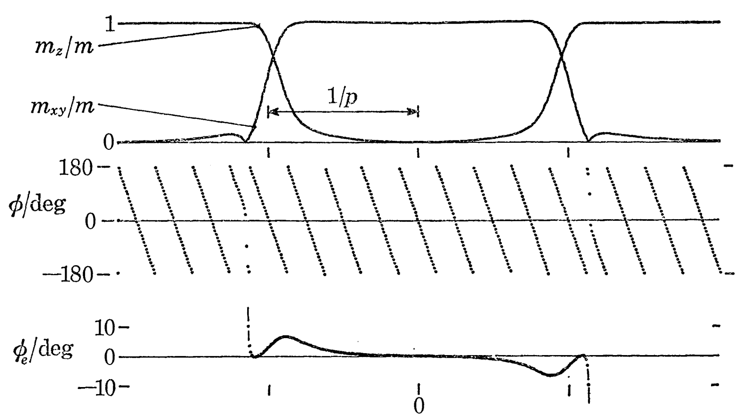

FIGURE 4. Magnetization at the end of the tailored pulse of figure 3 for $\theta = 90°$. The lower curve gives the remaining phase error at the centre of the echo.

The Bloch equations to be integrated can be written in the form

$$\Delta \boldsymbol{m} = (\boldsymbol{m} \times \gamma \boldsymbol{B}_{\mathrm{eff}})\, \Delta t. \tag{3}$$

The computer calculation was executed by division of the time scale into n time steps, Δt. In each step the increment $\Delta \boldsymbol{m}$ was calculated and then added to the existing $\boldsymbol{m}$ value. The number of steps, n, necessary for an accuracy of about 1% in the computed magnetization components was determined experimentally and turned out to be of the order of 10000. Rounding errors did not yet play a role. The instance of the rectangular r.f. pulse was taken as one of the tests of the computer program.

The total flip angle, θ, for spins resonating at the central frequency, $\omega/2\pi$, is a parameter in the program. For $\theta = 90°$ the computation result is given in figure 4. Note how dephased the

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spins are at the end of the tailored pulse, so that almost no signal will be observed. The phase angle, ϕ, however, is a fairly linear function of the frequency, at least within most of the region where spins are excited. This makes it possible to create an echo, for instance by reversing the gradient directly after the r.f. pulse.

At the point in time at which the echo has its maximum value, the remaining phase deviation, ϕ_e, is also calculated; this is plotted in figure 4. It varies only between -7 and $+7°$, in almost the entire region of appreciable spin excitation.

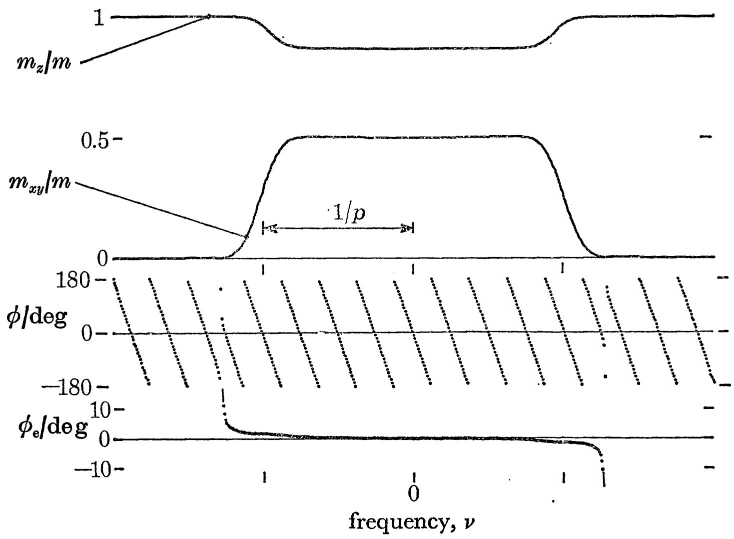

FIGURE 5. As figure 4, but for $\theta = 30°$.

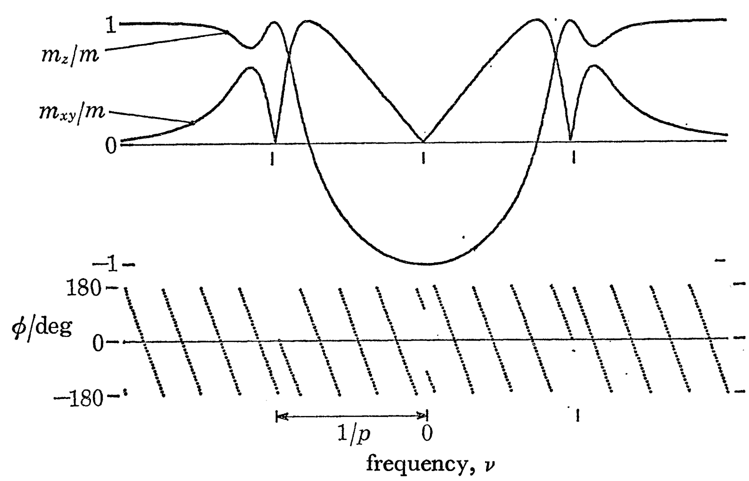

FIGURE 6. As figure 4, but for $\theta = 180°$.

The shape of m_{xy} clearly deviates, somewhat, from the Fourier transform in figure 3, but the deviation is still small.

These results are similar to those found by Hutchison *et al.* (1978). The response of the spin system is still roughly linear. The response is even more linear for $\theta = 30°$, as shown in figure 5.

This is in good agreement with the recent analysis given by Hoult (1979), who shows, by perturbation calculation, where the spin system starts to deviate from linearity.

The last instance shown is that of $\theta = 180°$ for the same tailored excitation function (equation (2)). Here we are primarily interested in the question of whether or not m_z is reversed in a selected region. In a large part of the region, the reversal is quite incomplete, as is shown in figure 6. Correspondingly, m_{xy} acquires large values near the edges of the region. If the relaxation times (T_1, T_2) are such that $T_2 \ll T_1$, one may let m_{xy} die away, but this condition is not met for all parts of the human body.

4. Some practical aspects

The tailored pulse described in § 3 (equation (2) and figure 3) has a full width, $t_w = 8p$ and excites a frequency range of full width, $\nu_w = 2/p$, so that $t_w \nu_w = 16$. If B_w is the full width of the corresponding region of the static magnetic field B_0, one has, for protons,

$$B_w t_w = 16 \times 2\pi/\gamma = 3.76 \times 10^{-4} \text{ T ms}.$$

If one wants to limit t_w to, say, 2 ms ($t_w \leqslant 2$ ms) so as not to lose too much time before detection can take place and not to have too much influence from unwanted fluctuations in magnetic field, one has $B_w \geqslant 1.38 \times 10^{-4}$ T. If the excited slice thickness has to be 1 cm, then one needs a gradient, G, $\geqslant 1.38 \times 10^{-4}$ T/cm, which is already quite difficult to obtain in large volumes.

For a coil around the body, the maximum r.f. power needed for the mentioned pulse of 2 ms length is calculated to be of the order of 200 W, which is not too difficult to achieve technically.

5. Conclusions

Selective excitation for large body spin imaging can very satisfactorily be achieved by use of an amplitude-modulated r.f. pulse, followed by a means of time reversal after the pulse, to bring the strongly dephased spins back into phase (echo formation), as is proposed by Hoult (1977) and by Hutchison *et al.* (1978). Whereas a rectangular, or approximately rectangular, r.f. pulse does not give a good selection, good results are obtained by means of an r.f. pulse that is the product of a sinc function and a Gauss function.

Our findings agree well with Hoult's (1979) analysis.

The complement of selective excitation, namely selective saturation, is still a topic for which it is desirable to do a precise analysis.

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